

Multicomponent theory of buoyancy instabilities in astrophysical plasma objects: The case of magnetic field perpendicular to gravity

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ABSTRACT

We develop a general theory of buoyancy instabilities in the electron-ion plasma with the electron heat flux based not upon MHD equations, but using a multicomponent plasma approach in which the momentum equation is solved for each species. We investigate the geometry in which the background magnetic field is perpendicular to the gravity and stratification. General expressions for the perturbed velocities are given without any simplifications. Collisions between electrons and ions are taken into account in the

momentum equations in a general form, permitting us to consider both weakly and strongly collisional objects. However, the electron heat flux is assumed to be directed along the magnetic field that implies a weakly collisional case. Using simplifications justified for an investigation of buoyancy instabilities with the electron thermal flux, we derive simple dispersion relations both for collisionless and collisional cases for arbitrary directions of the wave vector. The collisionless dispersion relation considerably differs from that obtained in the MHD framework and is similar to the Schwarzschild’s criterion. This difference is connected with simplified assumptions used in the MHD analysis of buoyancy instabilities and with the role of the longitudinal electric field perturbation which is not captured by the ideal MHD equations. The results obtained can be applied to clusters of galaxies and other astrophysical objects.

Subject headings: convection - instabilities - magnetic fields - plasmas - waves

1. INTRODUCTION

Physical processes taking place in astrophysical objects are defined by the physical parameters of the latter. In many cases, evolution of these parameters can lead to instabilities influencing the dynamics of these objects. Convective or buoyancy instabilities arising as a result of stratification of astrophysical objects are among those instabilities that may operate under different conditions from stellar interiors (e.g., Schwarzschild 1958), accretion disks (Balbus 2000, 2001), and neutron stars (Chang & Quataert 2009) to hot accretion flows (e.g., Narayan et al. 2000, 2002) and even galaxy clusters and intracluster medium (ICM) (e.g., Quataert 2008; Sharma et al. 2009; Ren et al. 2009). Analogous instabilities also exist in the neutral atmosphere of the Earth and ocean (Gossard & Hooke 1975; Pedlosky 1982). It is believed that convective instabilities have a vital role not only in transporting energy but in driving turbulence in many astrophysical systems. When the entropy increases in the direction of gravity, a thermally stratified fluid becomes buoyantly unstable according to the Schwarzschild criterion (Schwarzschild 1958). This well-known condition is successfully applied in the theoretical modeling of stellar structure and is valid irrespective of existence of the magnetic field. But a stellar fluid is a strong collisional system. At the same time, there are systems in which plasma is tenuous and hot such as, for example, ICM (Sarazin 1988). Majority of the mass of a galaxy cluster is in the dark matter. However, around 1/6 of its mass consists of hot, magnetized, and low density plasma known as ICM. The ICM is classified as a weakly collisional plasma with the electron number density $n_e \simeq 10^{-2} - 10^{-1} \text{ cm}^{-3}$, the electron temperature T_e of the order of a few keV (e.g., Fabian et al. 2006; Sanders et al. 2010), and the magnetic field strength $B \simeq 0.1 - 10 \text{ } \mu\text{G}$ (Carilli & Taylor 2002). Thus in ICM, the mean free path of ions and electrons is much larger than their Larmor radius. Cosmic rays play also an important role in the physics of ISM. Recent studies show that centrally concentrated cosmic rays

have a destabilizing effect on the convection in ICM (Chandran & Dennis 2006; Rasera and Chandran 2008).

In the recent past, one has included the anisotropic heat flux in weakly collisional plasmas and obtained additional instabilities. These instabilities have been shown to arise when the temperature increases in the direction of gravity, if the background thermal flux is absent (the magnetothermal instability (MTI)) (Balbus 2000, 2001), and when the temperature decreases along the gravity at the presence of the latter (the heat buoyancy instability (HBI)) (Quataert 2008). The growth rates of MTI and HBI are of the same order of magnitude as the growth rates without heat flux.

Previous theoretical models applied for study of buoyancy instabilities are based on the ideal magnetohydrodynamic (MHD) equations (Balbus 2000, 2001; Quataert 2008; Chang & Quataert 2009; Ren et al. 2009). However, the ideal MHD does not capture some important effects. One such effect is the nonzero longitudinal electric field perturbation along the magnetic field. We show here that the contribution of currents due to this small (at the present case) field to the dispersion relation can be of the same order of magnitude as that due to the electric field components transverse to the magnetic field. Besides, the MHD equations can not take into account the existence of various charged and neutral species with the different masses and electric charges and collisions of different species with each other. On the contrary, the plasma **E**-approach deals with dynamical equations for each species. Starting with Faraday’s and Ampere’s laws, one obtains equations for the electric field components. Such an approach allows us to follow the movement and change of parameters of each species separately and to obtain rigorous conditions of consideration and corresponding physical consequences in specific cases. This approach permits us to include various species of ions and dust grains having different charges and masses. In this way, streaming instabilities of rotating multicomponent objects with different background

velocities of species (accretion disks, molecular clouds and so on) have been investigated by Nekrasov (2008, 2009 a, b, c, d). It has been shown that these instabilities have the growth rates much larger than that of the magnetorotational instability (Balbus 1991). In some cases, the standard methods used in MHD leads to conclusions that significantly differ from those obtained by the method using the electric field perturbations. One of such examples concerning the contribution of collisions in the electron-ion plasma has been shown in (Nekrasov 2009 c).

A study of buoyancy instabilities with the electron heat flux by the multicomponent **E**-approach has been performed by Nekrasov and Shadmehri (2010). A geometry has been considered in which the gravity, stratification, and background magnetic field are all directed along the one axis. Solution of the dispersion relation has shown that the thermal flux has a stabilizing effect. The same problem solved by the ideal MHD leads to another result (Quataert 2008).

In this paper, we apply the multicomponent **E**-approach to study buoyancy instabilities in magnetized nonuniform electron-ion astrophysical plasmas in which the background magnetic field is perpendicular to the gravitational field. The inhomogeneity is assumed to be directed along the gravity. We include collisions between electrons and ions and electron thermal conductivity. The dynamical frequency and the wave phase velocity along the magnetic field are assumed much less than the ion cyclotron frequency and electron thermal velocity, respectively. In this case, the electrons can be regarded as inertialess. The pressure anisotropy of species is not taken into account. Thus we exclude the pressure-anisotropy-driven firehose (e.g., Vedenov & Sagdeev 1958; Kennel & Sagdeev 1967 a, b; Schekochihin et al. 2008) and mirror (e.g., Vedenov & Sagdeev 1959, Southwood & Kivelson 1993; Califano et al. 2008) mode instabilities. We also neglect the viscous and finite Larmor radius effects. The corresponding conditions can be easily obtained from

momentum equations given, for example, in (Nekrasov 2009 c, d). Solutions for perturbed velocities of species are found in a general form. Using conditions suitable for buoyancy perturbations, we obtain simplified expressions for perturbed values. The final dispersion relation is derived for the case in which the anisotropic thermal flux is important. The growth rates are found for collisionless as well as collisional cases. Solutions of the dispersion relation are discussed.

The paper is organized as follows. In Section 2, the fundamental equations are given. The equilibrium state is considered in Section 3. The perturbed ion velocity, number density, and thermal pressure are obtained in Section 4. In Section 5, we consider the perturbed velocity and temperature for electrons. Perturbed current components are calculated in Section 6. In Section 7, we consider conditions to simplify the contribution from collisions. The dispersion relation is derived in Section 8. In Section 9, the main points of our analysis are discussed. Implications of results obtained for galaxy clusters are considered in Section 10. Concluding remarks are given in Section 11.

2. BASIC EQUATIONS

We start with the following equations for ions:

$$\frac{\partial \mathbf{v}_i}{\partial t} + \mathbf{v}_i \cdot \nabla \mathbf{v}_i = -\frac{\nabla p_i}{m_i n_i} + \mathbf{g} + \frac{q_i}{m_i} \mathbf{E} + \frac{q_i}{m_i c} \mathbf{v}_i \times \mathbf{B} - \nu_{ie} (\mathbf{v}_i - \mathbf{v}_e), \quad (1)$$

the momentum equation,

$$\frac{\partial n_i}{\partial t} + \nabla \cdot n_i \mathbf{v}_i = 0, \quad (2)$$

the continuity equation, and

$$\frac{\partial p_i}{\partial t} + \mathbf{v}_i \cdot \nabla p_i + \gamma p_i \nabla \cdot \mathbf{v}_i = 0, \quad (3)$$

the pressure equation. The corresponding equations for the inertialess electrons are:

$$\mathbf{0} = -\frac{\nabla p_e}{n_e} + q_e \mathbf{E} + \frac{q_e}{c} \mathbf{v}_e \times \mathbf{B} - m_e \nu_{ei} (\mathbf{v}_e - \mathbf{v}_i), \quad (4)$$

$$\frac{\partial n_e}{\partial t} + \nabla \cdot n_e \mathbf{v}_e = 0, \quad (5)$$

$$\frac{\partial p_e}{\partial t} + \mathbf{v}_e \cdot \nabla p_e + \gamma p_e \nabla \cdot \mathbf{v}_e = -(\gamma - 1) \nabla \cdot \mathbf{q}_e, \quad (6)$$

$$\frac{\partial T_e}{\partial t} + \mathbf{v}_e \cdot \nabla T_e + (\gamma - 1) T_e \nabla \cdot \mathbf{v}_e = -(\gamma - 1) \frac{1}{n_e} \nabla \cdot \mathbf{q}_e, \quad (7)$$

where the last equation is the temperature equation and \mathbf{q}_e is the electron heat flux (Braginskii 1965). In Equations (1)-(7), q_j and m_j are the charge and mass of species $j = i, e$, \mathbf{v}_j is the hydrodynamic velocity, n_j is the number density, $p_j = n_j T_j$ is the thermal pressure, T_j is the temperature, ν_{ie} (ν_{ei}) is the collision frequency of ions (electrons) with electrons (ions), \mathbf{g} is the gravity, \mathbf{E} and \mathbf{B} are the electric and magnetic fields, c is the speed of light in vacuum, and γ is the adiabatic constant. As for the electron heat flux, we consider the case of weakly collisional plasma for which the electron cyclotron frequency $\omega_{ce} = q_e B / m_e c$ is much larger than the electron-electron collision frequency ν_{ee} , i.e. $\omega_{ce} \gg \nu_{ee}$. In this case, the electron thermal flux is mainly directed along the magnetic field,

$$\mathbf{q}_e = -\chi_e \mathbf{b} (\mathbf{b} \cdot \nabla) T_e, \quad (8)$$

where χ_e is the electron thermal conductivity coefficient and $\mathbf{b} = \mathbf{B}/B$ is the unit vector along the magnetic field. We only take into account the electron thermal flux (8) because the ion thermal conductivity is considerably smaller (Braginskii 1965). We also assume that the thermal flux in equilibrium is negligible.

Electromagnetic equations are Faraday's

$$\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}, \quad (9)$$

and Ampere's

$$\nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{j}, \quad (10)$$

laws, where $\mathbf{j} = \sum_j q_j n_j \mathbf{v}_j$. We consider wave processes with typical time-scales much larger than the time the light spends to cover the wavelength of perturbations. In this case, one can neglect the displacement current in Equation (10) that results in quasi-neutrality for both the electromagnetic and purely electrostatic perturbations. The magnetic field \mathbf{B} includes the background magnetic field \mathbf{B}_0 , the magnetic field \mathbf{B}_{0cur} of the background electric current (when it presents), and the perturbed magnetic field.

Our basic equations are the same as that in (Nekrasov & Shadmehri 2010). However, in the present case, the equilibrium state differs from the equilibrium in (Nekrasov & Shadmehri 2010), where magnetic field lines and the gravity are parallel to each other. In the next section, we analyze the equilibrium state in which the magnetic field is perpendicular to the direction of gravity.

3. EQUILIBRIUM

3.1. Background velocities

At first, we consider the equilibrium state. In this paper, we study the configuration in which the background magnetic field \mathbf{B}_0 is parallel to the z -axis, and the gravity and stratification are parallel to the x -axis. Let, for definiteness, \mathbf{g} be $\mathbf{g} = -\mathbf{x}g$, where $g > 0$ and \mathbf{x} is the unit vector along the x -direction. Then, Equations (1) and (4) are the following:

$$\mathbf{v}_{i0} \cdot \nabla \mathbf{v}_{i0} = -\frac{\nabla p_{i0}}{m_i n_{i0}} + \mathbf{g} + \frac{q_i}{m_i} \mathbf{E}_0 + \frac{q_i}{m_i c} \mathbf{v}_{i0} \times \mathbf{B}_0 - \nu_{ie}^0 (\mathbf{v}_{i0} - \mathbf{v}_{e0}), \quad (11)$$

$$\mathbf{0} = -\frac{\nabla p_{e0}}{n_{e0}} + q_e \mathbf{E}_0 + \frac{q_e}{c} \mathbf{v}_{e0} \times \mathbf{B}_0 - m_e \nu_{ei}^0 (\mathbf{v}_{e0} - \mathbf{v}_{i0}), \quad (12)$$

where the index 0 denotes the background values. Our system is uniform along the y - and z -axes. Therefore, we assume that $E_{0y} = E_{0z} = 0$. However, for generality, we keep $E_{0x} \neq 0$. We also assume that $v_{i0z} = v_{e0z} = 0$. Then, from the y -components of Equations (11) and (12), we obtain

$$v_{i0x} = v_{e0x} = -\frac{\nu_{ie}^0}{\omega_{ci}}(v_{i0y} - v_{e0y}) \quad (13)$$

provided that $\omega_{ci} = q_i B_0 / m_i c \gg \partial v_{i0y} / \partial x$. In this case, the contribution of the ion inertia along the y -axis produced by the shear of velocity v_{i0y} along the x -axis can be neglected in comparison with the y -component of the Lorentz force. From the x -components of Equations (11) and (12), we also find under condition $\omega_{ci} v_{i0y} \gg \partial v_{i0x}^2 / 2 \partial x$

$$v_{i0y} = v_E + v_g + v_{di} = -c \frac{E_{0x}}{B_0} + \frac{g}{\omega_{ci}} - \frac{g_i}{\omega_{ci}}, \quad (14)$$

$$v_{e0y} = v_E + v_{de} = -c \frac{E_{0x}}{B_0} + \frac{g_e}{\omega_{ci}}, \quad (15)$$

where

$$g_i = -\frac{1}{m_i n_{i0}} \frac{\partial p_{i0}}{\partial x}, g_e = -\frac{1}{m_i n_{e0}} \frac{\partial p_{e0}}{\partial x}. \quad (16)$$

The velocities v_E , v_g , and $v_{di,e}$ are the electric, gravitational, and diamagnetic drifts, respectively. We assume that $q_i = -q_e$. If values g_i and g_e are inhomogeneous and $g_i \sim g_e$, then conditions given above to justify solutions (13) and (14) can be written in the following form:

$$1 \gg \left(1 + \frac{\nu_{ie}^{02}}{\omega_{ci}^2}\right) \frac{\rho_i^2}{L^2}, \quad (17)$$

where $\rho_i = v_{Ti} / \omega_{ci}$ (v_{Ti} is the ion thermal velocity) is the ion Larmor radius and $L^{-1} = \partial g_{i,e} / g_{i,e} \partial x$ (an uncertain contribution of the electric drift is not taken into account in condition [17]). We note that there are no restrictions on the relation between ω_{ci} and ν_{ie}^0 in inequality (17), i.e. the case $\nu_{ie}^0 > \omega_{ci}$ can also take place.

3.2. Continuity and pressure equations

Having the background velocities, we can now consider the ion continuity equation (2). In equilibrium, we have

$$\frac{\partial n_{i0}}{\partial t} + \frac{\partial}{\partial x} n_{i0} v_{i0x} = 0. \quad (18)$$

This equation is nonstationary because of the x -dependence of v_{i0x} . The typical time t_0 of the density evolution is $t_0 \sim 1/\nu_{ie}^0 \kappa^2 \rho_i^2$, where $\kappa = \partial n_0 / n_0 \partial x$ ($n_{i0} = n_{e0} = n_0$). When studying the linear perturbations, we will not take into account that the medium is nonstationary. This means that we will consider perturbations which develop much faster than t_0 . Thus, we can safely assume that the medium is stationary. The same also relates to the ion pressure equation and corresponding equations for electrons.

4. LINEAR ION PERTURBATIONS

4.1. Equation for the perturbed ion velocity

Let us write the equation of motion (1) for ions in the linear approximation,

$$\begin{aligned} \frac{\partial \mathbf{v}_{i1}}{\partial t} + \mathbf{v}_{i0} \cdot \nabla \mathbf{v}_{i1} + \mathbf{v}_{i1} \cdot \nabla \mathbf{v}_{i0} = & -\frac{\nabla p_{i1}}{m_i n_{i0}} + \frac{\nabla p_{i0}}{m_i n_{i0}} \frac{n_{i1}}{n_{i0}} + \frac{q_i}{m_i} \mathbf{E}_1 + \frac{q_i}{m_i c} \mathbf{v}_{i1} \times \mathbf{B}_0 \\ & + \frac{q_i}{m_i c} \mathbf{v}_{i0} \times \mathbf{B}_1 - \nu_{ie}^0 (\mathbf{v}_{i1} - \mathbf{v}_{e1}) - \nu_{ie}^1 (\mathbf{v}_{i0} - \mathbf{v}_{e0}), \end{aligned} \quad (19)$$

where the index 1 denotes the values of the first order of magnitude. The diamagnetic drift in the stationary velocity $\mathbf{v}_{i,e0}$ is not a real velocity (e.g. Nishikawa & Wakatani 1990). However, this drift must be taken into account in the hydrodynamical equations evoking drift waves (e.g. Vranješ et al. 2003 for the same initial state). The last collisional term in Equation (19) appears as a result of perturbation of collision frequency due to the density

and temperature perturbations: $\nu_{ie}^1/\nu_{ie}^0 = n_{e1}/n_{e0} - 3T_{e1}/2T_{e0}$. This effect due to the density perturbation has been involved by Nekrasov (e.g. 2009 b).

Below, we do not include the shear of the background velocity on the left hand-side of Equation (19) (the same also relates to the continuity and pressure equations). It is possible, if

$$D_{ti} = \frac{\partial}{\partial t} + \mathbf{v}_{i0} \cdot \nabla \gg \frac{\partial v_{i0x}}{\partial x}; \frac{\partial v_{i0y}}{\partial x}. \quad (20)$$

The first inequality (20), $D_{ti} \gg \partial v_{i0x}/\partial x$, coincides with the condition of medium stationarity (see Eq. [18]). The second inequality, $D_{ti} \gg \partial v_{i0y}/\partial x$, is satisfied automatically if $v_{i0y} = \text{const}$. When $v_{i0y} \neq \text{const}$, this condition can be written in the form $D_{ti} \gg \omega_{ci} \rho_i^2 / L^2$. We further introduce the following notations:

$$\begin{aligned} \mathbf{F}_{i1} &= \frac{q_i}{m_i} \mathbf{E}_1 - \nu_{ie}^0 (\mathbf{v}_{i1} - \mathbf{v}_{e1}), \\ \mathbf{G}_{i1} &= \mathbf{F}_{i1} + \frac{q_i}{m_i c} \mathbf{v}_{i0} \times \mathbf{B}_1, \\ \mathbf{C}_{i1} &= \nu_{ie}^1 (\mathbf{v}_{i0} - \mathbf{v}_{e0}). \end{aligned} \quad (21)$$

Then Equation (19) takes the form

$$D_{ti} \mathbf{v}_{i1} = - \frac{\nabla p_{i1}}{m_i n_{i0}} + \frac{\nabla p_{i0}}{m_i n_{i0}} \frac{n_{i1}}{n_{i0}} + \mathbf{G}_{i1} - \mathbf{C}_{i1} + \frac{q_i}{m_i c} \mathbf{v}_{i1} \times \mathbf{B}_0. \quad (22)$$

4.2. Perturbed ion continuity and pressure equations

In the linear approximation, the ion continuity (Eq. [2]) and pressure (Eq. [3]) equations are given by

$$D_{ti} n_{i1} + v_{i1x} \frac{\partial n_{i0}}{\partial x} + n_{i0} \nabla \cdot \mathbf{v}_{i1} = 0, \quad (23)$$

$$D_{ti}p_{i1} + v_{i1x} \frac{\partial p_{i0}}{\partial x} + \gamma p_{i0} \nabla \cdot \mathbf{v}_{i1} = 0. \quad (24)$$

Equations (22)-(24) are used to find perturbed ion velocity components. These calculations are given in the Appendix A. For simplicity, we adopt that $g_{i,e} = \text{const}$ (see Eq. [16]).

This is true when the gravity is directed along the background magnetic field (Nekrasov & Shadmehri 2010). This case is also satisfied, if we impose the condition $v_{i,e1y} = 0$ (see Eqs. [14] and [15]). However, in a general case $g_{i,e} \neq \text{const}$.

5. LINEAR ELECTRON PERTURBATIONS

5.1. Equation for the perturbed electron velocity

Consider Equation (4) in the linear approximation

$$\mathbf{0} = -\frac{\nabla p_{e1}}{n_{e0}} + \frac{\nabla p_{e0}}{n_{e0}} \frac{n_{e1}}{n_{e0}} + q_e \mathbf{E}_1 + \frac{q_e}{c} \mathbf{v}_{e1} \times \mathbf{B}_0 + \frac{q_e}{c} \mathbf{v}_{e0} \times \mathbf{B}_1 - m_e \nu_{ei}^0 (\mathbf{v}_{e1} - \mathbf{v}_{i1}) - m_e \nu_{ei}^1 (\mathbf{v}_{e0} - \mathbf{v}_{i0}). \quad (25)$$

Here $\nu_{ei}^1/\nu_{ei}^0 = n_{i1}/n_{i0} - 3T_{e1}/2T_{e0}$. We introduce the following notations:

$$\mathbf{F}_{e1} = q_e \mathbf{E}_1 - m_e \nu_{ei}^0 (\mathbf{v}_{e1} - \mathbf{v}_{i1}), \quad (26)$$

$$\mathbf{G}_{e1} = \mathbf{F}_{e1} + \frac{q_e}{c} \mathbf{v}_{e0} \times \mathbf{B}_1,$$

$$\mathbf{C}_{e1} = m_e \nu_{ei}^1 (\mathbf{v}_{e0} - \mathbf{v}_{i0}).$$

Then Equation (25) takes the form

$$\mathbf{0} = -\frac{\nabla p_{e1}}{n_{e0}} + \frac{\nabla p_{e0}}{n_{e0}} \frac{n_{e1}}{n_{e0}} + \mathbf{G}_{e1} - \mathbf{C}_{e1} + \frac{q_e}{c} \mathbf{v}_{e1} \times \mathbf{B}_0. \quad (27)$$

5.2. Perturbed electron continuity, pressure, and temperature equations

The continuity equation (5) in the linear approximation is given by

$$D_{te}n_{e1} + v_{e1x}\frac{\partial n_{e0}}{\partial x} + n_{e0}\nabla \cdot \mathbf{v}_{e1} = 0, \quad (28)$$

where $D_{te} = \partial/\partial t + \mathbf{v}_{e0} \cdot \nabla$. Here, we also neglect the contribution of equilibrium velocity inhomogeneity, i.e. we assume that $D_{te} \gg \partial v_{e0x}/\partial x$.

The linear electron pressure equation (6) is the following:

$$D_{te}p_{e1} + v_{e1x}\frac{\partial p_{e0}}{\partial x} + \gamma p_{e0}\nabla \cdot \mathbf{v}_{e1} + (\gamma - 1)\nabla \cdot \mathbf{q}_{e1} = 0, \quad (29)$$

where \mathbf{q}_{e1} is the linear electron thermal flux.

The perturbed electron temperature equation (7) has the form

$$D_{te}T_{e1} + v_{e1x}\frac{\partial T_{e0}}{\partial x} + (\gamma - 1)T_{e0}\nabla \cdot \mathbf{v}_{e1} = -(\gamma - 1)\frac{1}{n_{e0}}\nabla \cdot \mathbf{q}_{e1}. \quad (30)$$

5.3. Perturbed electron thermal flux

Expression for the thermal flux (8) in the linear approximation is

$$\mathbf{q}_{e1} = -\chi_{e0}\mathbf{b}_0(\mathbf{b}_1 \cdot \nabla)T_{e0} - \chi_{e0}\mathbf{b}_0\frac{\partial T_{e1}}{\partial z}, \quad (31)$$

where

$$\mathbf{b}_1 = \frac{\mathbf{B}_1}{B_0} - \mathbf{b}_0\frac{B_{1z}}{B_0}.$$

Thus from Equation (31), we have

$$\begin{aligned} q_{e1x,y} &= 0, \\ q_{e1z} &= -\chi_{e0}\frac{\partial T_{e0}}{\partial x}\frac{B_{1x}}{B_0} - \chi_{e0}\frac{\partial T_{e1}}{\partial z}. \end{aligned} \quad (32)$$

Using Equation (32), we find the value $\nabla \cdot \mathbf{q}_{e1}$ as follows

$$\nabla \cdot \mathbf{q}_{e1} = -\chi_{e0} \frac{\partial T_{e0}}{\partial x} \frac{1}{B_0} \frac{\partial B_{1x}}{\partial z} - \chi_{e0} \frac{\partial^2 T_{e1}}{\partial z^2}. \quad (33)$$

The first term on the right hand-side of Equation (33) can be considered as a source of the heat, and the second term describes the thermal diffusion. This expression for $\nabla \cdot \mathbf{q}_{e1}$ is analogous to the one arising in the case in which the stratification is directed along the background magnetic field and there exists an anisotropic background thermal flux (see Nekrasov & Shadmehri 2010, Eq. [49]).

5.4. Perturbed electron temperature and pressure equations with the thermal flux

Equation (30), taking into account expression (33), can be written in the form

$$(D_{te} + \Omega) T_{e1} = -v_{e1x} \frac{\partial T_{e0}}{\partial x} - (\gamma - 1) T_{e0} \nabla \cdot \mathbf{v}_{e1} - \frac{\partial T_{e0}}{\partial x} \frac{1}{B_0} \Omega \left(\frac{\partial}{\partial z} \right)^{-1} B_{1x}, \quad (34)$$

where the operator

$$\Omega = -(\gamma - 1) \frac{1}{n_{e0}} \chi_{e0} \frac{\partial^2}{\partial z^2} \quad (35)$$

is introduced.

Equation (34) permits us to define the value $\nabla \cdot \mathbf{q}_{e1}$. Substituting this value in Equation (29), we obtain the desirable equation for the perturbed electron pressure

$$\begin{aligned} 0 = & D_{te} p_{e1} + \left[\frac{\partial p_{e0}}{\partial x} - n_{e0} \frac{\Omega}{(D_{te} + \Omega)} \frac{\partial T_{e0}}{\partial x} \right] v_{e1x} + p_{e0} \frac{\gamma D_{te} + \Omega}{(D_{te} + \Omega)} \nabla \cdot \mathbf{v}_{e1} \\ & + n_{e0} \frac{\partial T_{e0}}{\partial x} \frac{1}{B_0} \frac{D_{te} \Omega}{(D_{te} + \Omega)} \left(\frac{\partial}{\partial z} \right)^{-1} B_{1x}. \end{aligned} \quad (36)$$

Equations (27), (28), and (36) are used in the Appendix B for determining the velocity \mathbf{v}_{e1} . For convenience and compactness of all the equations and expressions, we formally apply the method of division by time and spatial derivatives (see Eqs. [34], [36], and below). This permits us not to use the Fourier transformation at the initial stage of calculations, but rather gives a possibility to obtain the spatiotemporal differential equations for variables under study (for example, for numerical calculations). When the dispersion relation is derived in Section 8, all derivatives will be changed by their Fourier-images.

6. CURRENT \mathbf{j}_1

Expressions for \mathbf{v}_{i1} and \mathbf{v}_{e1} obtained in the Appendices A and B have a sufficiently general form. In particular, one can take into account the background velocities of species that results in electromagnetic streaming instabilities. Some studies of such instabilities in astrophysical objects are given in Nekrasov (2007; 2008; 2009 a,b,c,d). However, in the present paper, we do not consider this effect, assuming that

$$\begin{aligned} D_t &\gg \mathbf{v}_{i,e0} \cdot \nabla, \\ D_t \mathbf{E}_1 &\gg \nabla \mathbf{v}_{i,e0} \cdot \mathbf{E}_1, \end{aligned} \tag{37}$$

where $D_t = \partial/\partial t$. Thus, $D_{ti,e} \simeq D_t$. The second inequality (37) is obtained from condition $c\mathbf{E}_1 \gg \mathbf{v}_{i,e0} \times \mathbf{B}_1$ (see Eqs. [21] and [26]) and by using Equation (9). Under conditions (37), one can also neglect the contribution of $\mathbf{C}_{i,e1}$ in Equations (22) and (27). We further consider perturbations with the dynamical frequency much smaller than the ion sound frequency. Keeping the spatiotemporal derivatives, this means that $c_{si}^2 \partial^2/\partial z^2 \gg D_t^2$. We also assume a local approximation when the perturbation wavelength in the direction of inhomogeneity is smaller than the inhomogeneity scale, i.e. $\partial/\partial x \gg g_{i,e}/c_{si,e}^2$. Below, we find components of the perturbed current \mathbf{j}_1 . For simplicity, we omit the index 0 by ν_{ie}^0 .

6.1. Current j_{1x}

From Equation (A15) and under conditions given above, we obtain

$$\begin{aligned} v_{i1x} = & \frac{D_t}{\omega_{ci}^2} \left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \left(\frac{\partial}{\partial z} \right)^{-2} F_{i1x} + \frac{1}{\omega_{ci}} F_{i1y} - \frac{D_{ti}}{\omega_{ci}^2} \frac{\partial^2}{\partial x \partial y} \left(\frac{\partial}{\partial z} \right)^{-2} F_{i1y} \\ & - \frac{1}{\omega_{ci}} \frac{\partial}{\partial y} \left(\frac{\partial}{\partial z} \right)^{-1} F_{i1z} - \frac{D_t}{\omega_{ci}^2} \left[\frac{\omega_{ci} D_t}{c_{si}^2} \frac{\partial}{\partial y} \left(\frac{\partial}{\partial z} \right)^{-2} + \frac{\partial}{\partial x} \right] \left(\frac{\partial}{\partial z} \right)^{-1} F_{i1z}, \end{aligned} \quad (38)$$

where the additional condition $\partial/\partial x \gg (g_i/c_{si}^2) (\partial/\partial y)^2 (\partial/\partial z)^{-2}$ has been assumed. From Equation (B4), we have

$$v_{e1x} = \frac{1}{m_e \omega_{ce}} F_{e1y} - \frac{1}{m_e \omega_{ce}} \frac{\partial}{\partial y} \left(\frac{\partial}{\partial z} \right)^{-1} F_{e1z}. \quad (39)$$

Expression (38) coincides with expression (39) in the limit $m_i \rightarrow 0$ at substitution $i \leftrightarrow e$ and $m_i \mathbf{F}_{i1} \rightarrow \mathbf{F}_{e1}$.

Using expressions (38) and (39), we can find the current j_{1x} . It is convenient to find the value $4\pi j_{1x}/D_t$. As a result, we obtain

$$\frac{4\pi}{D_t} \left(1 + \frac{D_t \nu_{ie}}{\omega_{pi}^2} a_{xx} \right) j_{1x} = a_{xx} E_{1x} - a_{xy} E_{1y} - a_{xz} E_{1z} + \frac{4\pi \nu_{ie}}{\omega_{pi}^2} a_{xy} j_{1y} + \frac{4\pi \nu_{ie}}{\omega_{pi}^2} a_{xz} j_{1z}. \quad (40)$$

The following notations are introduced here:

$$\begin{aligned} a_{xx} &= \frac{\omega_{pi}^2}{\omega_{ci}^2} \left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \left(\frac{\partial}{\partial z} \right)^{-2}, \quad a_{xy} = \frac{\omega_{pi}^2}{\omega_{ci}^2} \frac{\partial^2}{\partial x \partial y} \left(\frac{\partial}{\partial z} \right)^{-2}, \\ a_{xz} &= \frac{\omega_{pi}^2}{\omega_{ci}^2} \left[\frac{\omega_{ci} D_t}{c_{si}^2} \frac{\partial}{\partial y} \left(\frac{\partial}{\partial z} \right)^{-2} + \frac{\partial}{\partial x} \right] \left(\frac{\partial}{\partial z} \right)^{-1}. \end{aligned} \quad (41)$$

When deriving Equation (40), we have used expressions (21) and (26) for \mathbf{F}_{i1} and \mathbf{F}_{e1} , respectively. We note that in Equation (40) and below in this section, we have not imposed any restrictions on the collision frequency ν_{ie} in comparison with the ion cyclotron frequency ω_{ci} .

6.2. Current j_{1y}

Let us consider the velocity v_{i1y} . Using corresponding conditions, we can write from Equation (A18)

$$\begin{aligned}
 v_{i1y} = & -\frac{1}{\omega_{ci}} F_{i1x} - \frac{D_t}{\omega_{ci}^2} \frac{\partial^2}{\partial x \partial y} \left(\frac{\partial}{\partial z} \right)^{-2} F_{i1x} \\
 & + \frac{1}{D_t \omega_{ci}^2} \left[D_t^2 \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) \left(\frac{\partial}{\partial z} \right)^{-2} + \omega_{bi}^2 \right] F_{i1y} \\
 & + \frac{1}{\omega_{ci}} \frac{\partial}{\partial x} \left(\frac{\partial}{\partial z} \right)^{-1} F_{i1z} + \frac{1}{\omega_{ci}} \left[\frac{D_t^2}{c_{si}^2} \frac{\partial}{\partial x} \left(\frac{\partial}{\partial z} \right)^{-2} - \frac{\omega_{bi}^2}{g_i} \right] \left(\frac{\partial}{\partial z} \right)^{-1} F_{i1z} \\
 & - \frac{1}{D_t \omega_{ci}^2} (D_t^2 + \omega_{bi}^2) \frac{\partial}{\partial y} \left(\frac{\partial}{\partial z} \right)^{-1} F_{i1z}.
 \end{aligned} \tag{42}$$

Here, we have assumed that $\partial/\partial x \gg (g_i/c_{si}^2) (\partial/\partial x)^2 (\partial/\partial z)^{-2}$. We keep the last term in Equation (42) because the term proportional to $(\omega_{bi}^2/\omega_{ci} g_i) F_{i1z}$ can be canceled with the corresponding electron term (see below). Here and below, we have introduced the notation

$$\omega_{bi,e}^2 = \frac{g_{i,e}}{c_{si,e}^2} \left[g_{i,e} (\gamma - 1) + \frac{\partial c_{si,e}^2}{\partial x} \right], \tag{43}$$

which may be named the ion (electron) Brunt-Väisälä frequency.

The velocity v_{e1y} is defined by Equation (B6). For the case under consideration, we obtain

$$\begin{aligned}
 m_e \omega_{ce} v_{e1y} = & -F_{e1x} + m_i \omega_{be}^2 \frac{\gamma}{\gamma D_t + \Omega} \frac{1}{m_e \omega_{ce}} F_{e1y} + \frac{\partial}{\partial x} \left(\frac{\partial}{\partial z} \right)^{-1} F_{e1z} \\
 & - m_i \omega_{be}^2 \frac{\gamma}{\gamma D_t + \Omega} \frac{1}{m_e \omega_{ce}} \frac{\partial}{\partial y} \left(\frac{\partial}{\partial z} \right)^{-1} F_{e1z} \\
 & - \frac{1}{c_{se}^2} \left[(\gamma - 1) g_e \frac{\gamma D_t}{\gamma D_t + \Omega} + \frac{\partial c_{se}^2}{\partial x} \right] \left(\frac{\partial}{\partial z} \right)^{-1} F_{e1z} \\
 & + g_e m_i \frac{\partial T_{e0}}{T_{e0} \partial x} \frac{1}{B_0} \frac{\Omega}{\gamma D_t + \Omega} \left(\frac{\partial}{\partial z} \right)^{-1} B_{1x}.
 \end{aligned} \tag{44}$$

If we put $\Omega = 0$ in Equation (44) and take $m_i \rightarrow 0$ in Equation (42), we obtain the full conformity of both equations with each other.

Using expressions (42) and (44), we find the current j_{1y} . Taking into account (21) and (26), we obtain

$$\begin{aligned} \frac{4\pi}{D_t} \left(1 + \frac{D_t \nu_{ie}}{\omega_{pi}^2} a_{yy} \right) j_{1y} &= -a_{yx} E_{1x} + (a_{yy} + a_{yy}^*) E_{1y} + (a_{yz} - a_{yz}^*) E_{1z} \\ &+ \frac{4\pi \nu_{ie}}{\omega_{pi}^2} a_{yx} j_{1x} - \frac{4\pi \nu_{ie}}{\omega_{pi}^2} a_{yz} j_{1z}, \end{aligned} \quad (45)$$

where the following notations are introduced:

$$\begin{aligned} a_{yx} &= \frac{\omega_{pi}^2}{\omega_{ci}^2} \frac{\partial^2}{\partial x \partial y} \left(\frac{\partial}{\partial z} \right)^{-2}, \quad a_{yy} = \frac{\omega_{pi}^2}{\omega_{ci}^2} \left[\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) \left(\frac{\partial}{\partial z} \right)^{-2} + \frac{\omega_{bi}^2}{D_t^2} + \frac{\omega_{be}^2}{D_t^2} \frac{\gamma D_t}{\gamma D_t + \Omega} \right] \\ a_{yz} &= \frac{\omega_{pi}^2}{D_t \omega_{ci}} \left\{ \frac{D_t^2}{c_{si}^2} \frac{\partial}{\partial x} \left(\frac{\partial}{\partial z} \right)^{-2} - \frac{\omega_{bi}^2}{g_i} + \frac{1}{c_{se}^2} \left[(\gamma - 1) g_e \frac{\gamma D_t}{\gamma D_t + \Omega} + \frac{\partial c_{se}^2}{\partial x} \right] \right\} \left(\frac{\partial}{\partial z} \right)^{-1} \\ &- \frac{\omega_{pi}^2}{\omega_{ci}^2} \left\{ 1 + \frac{\omega_{bi}^2}{D_t^2} + \frac{\gamma \omega_{be}^2}{D_t (\gamma D_t + \Omega)} \right\} \frac{\partial}{\partial y} \left(\frac{\partial}{\partial z} \right)^{-1}, \\ a_{yy}^* &= \frac{1}{D_t^2} \frac{\omega_{pi}^2}{\omega_{ci}^2} g_e \frac{\partial T_{e0}^*}{T_{e0} \partial x} \frac{\Omega}{\gamma D_t + \Omega}, \quad a_{yz}^* = a_{yy}^* \frac{\partial}{\partial y} \left(\frac{\partial}{\partial z} \right)^{-1}. \end{aligned} \quad (46)$$

The terms proportional to $a_{yy,z}^*$ are connected with the contribution of the heat flux $\sim B_{1x}$ (see Eq. [32]). This magnetic field perturbation has been expressed via electric field perturbation through Equation (9).

We see from the expression for a_{yz} that for $\Omega = 0$ and $T_{i0} = T_{e0}$ the last two terms in the first figured brackets are canceled.

6.3. Current j_{1z}

From Equation (A20), we find the simplified ion velocity v_{i1z}

$$\begin{aligned} \frac{\partial v_{i1z}}{\partial z} &= \frac{1}{\omega_{ci}} \frac{\partial F_{i1x}}{\partial y} + \frac{D_t^2}{\omega_{ci} c_{si}^2} \frac{\partial}{\partial y} \left(\frac{\partial}{\partial z} \right)^{-2} F_{i1x} - \frac{D_t}{\omega_{ci}^2} \frac{\partial F_{i1x}}{\partial x} - \frac{1}{\omega_{ci}} \frac{\partial F_{i1y}}{\partial x} + \frac{g_i}{\omega_{ci} c_{si}^2} F_{i1y} \\ &- \frac{D_t^2}{\omega_{ci} c_{si}^2} \frac{\partial}{\partial x} \left(\frac{\partial}{\partial z} \right)^{-2} F_{i1y} - \frac{1}{D_t \omega_{ci}^2} (D_t^2 + \omega_{bi}^2) \frac{\partial F_{i1y}}{\partial y} - \frac{D_t}{c_{si}^2} \left(\frac{\partial}{\partial z} \right)^{-1} F_{i1z}. \end{aligned} \quad (47)$$

We have taken into account that $\partial/\partial x \gg (g_i/c_{si}^2) [(\partial/\partial x)^2 + (\partial/\partial y)^2] (\partial/\partial z)^{-2}$. From Equation (B8), we have

$$\begin{aligned} \frac{\partial v_{e1z}}{\partial z} = & \frac{1}{m_e \omega_{ce}} \frac{\partial F_{e1x}}{\partial y} - \frac{1}{m_e \omega_{ce}} \frac{\partial F_{e1y}}{\partial x} + \frac{1}{c_{se1}^2} \left[g_e + \frac{\Omega}{(D_t + \Omega)} \frac{\partial T_{e0}}{m_i \partial x} \right] \frac{1}{m_e \omega_{ce}} F_{e1y} \quad (48) \\ & - \frac{\gamma \omega_{be}^2 m_i}{(\gamma D_t + \Omega)} \left(\frac{1}{m_e \omega_{ce}} \right)^2 \frac{\partial F_{e1y}}{\partial y} - \frac{D_t}{c_{se1}^2 m_i} \left(\frac{\partial}{\partial z} \right)^{-1} F_{e1z} \\ & - \frac{\partial T_{e0}}{c_{se1}^2 m_i \partial x} \frac{1}{B_0} \frac{D_t \Omega}{(D_t + \Omega)} \left(\frac{\partial}{\partial z} \right)^{-1} B_{1x}, \end{aligned}$$

where

$$c_{se1}^2 = c_{se}^2 \frac{(\gamma D_{te} + \Omega)}{\gamma (D_{te} + \Omega)}. \quad (49)$$

If we take in Equation (47) $m_i \rightarrow 0$ and put $\Omega = 0$ in Equation (48), we obtain the conformity of these two equations.

Using Equations (9), (47), and (48), we find the longitudinal current j_{1z}

$$\begin{aligned} \frac{4\pi}{D_t} \left(1 - \frac{D_t \nu_{ie}}{\omega_{pi}^2} a_{zz} \right) j_{1z} = & a_{zx} E_{1x} + (a_{zy} + a_{zy}^*) E_{1y} - (a_{zz} + a_{zz}^*) E_{1z} - \frac{4\pi \nu_{ie}}{\omega_{pi}^2} a_{zx} j_{1x} \quad (50) \\ & - \frac{4\pi \nu_{ie}}{\omega_{pi}^2} a_{zy} j_{1y}. \end{aligned}$$

The following notations are introduced here:

$$\begin{aligned} a_{zx} = & \frac{\omega_{pi}^2}{\omega_{ci}^2} \left[\frac{\omega_{ci} D_t}{c_{si}^2} \frac{\partial}{\partial y} \left(\frac{\partial}{\partial z} \right)^{-2} - \frac{\partial}{\partial x} \right] \left(\frac{\partial}{\partial z} \right)^{-1}, a_{zz} = \omega_{pi}^2 \left(\frac{1}{c_{si}^2} + \frac{1}{c_{se1}^2} \right) \left(\frac{\partial}{\partial z} \right)^{-2}, \quad (51) \\ a_{zy} = & \frac{\omega_{pi}^2}{\omega_{ci} D_t} \left\{ \frac{g_i}{c_{si}^2} - \frac{1}{c_{se1}^2} \left[g_e + \frac{\Omega}{(D_t + \Omega)} \frac{\partial T_{e0}}{m_i \partial x} \right] - \frac{D_t^2}{c_{si}^2} \frac{\partial}{\partial x} \left(\frac{\partial}{\partial z} \right)^{-2} \right\} \left(\frac{\partial}{\partial z} \right)^{-1} \\ & - \frac{\omega_{pi}^2}{\omega_{ci}^2 D_t^2} \left(D_t^2 + \omega_{bi}^2 + \omega_{be}^2 \frac{\gamma D_t}{\gamma D_t + \Omega} \right) \frac{\partial}{\partial y} \left(\frac{\partial}{\partial z} \right)^{-1}, \\ a_{zy}^* = & \frac{\omega_{pi}^2}{\omega_{ci} D_t} \frac{\partial T_{e0}}{c_{se1}^2 m_i \partial x} \frac{\Omega}{(D_t + \Omega)} \left(\frac{\partial}{\partial z} \right)^{-1}, a_{zz}^* = a_{zy}^* \frac{\partial}{\partial y} \left(\frac{\partial}{\partial z} \right)^{-1}. \end{aligned}$$

The terms proportional to $a_{zy,z}^*$ in Equation (50) are connected with the contribution of the electron current induced by B_{1x} (see Eq. [48]).

7. SIMPLIFIED COLLISION CONTRIBUTION

To take into account collisions between ions and electrons, we involve some simplifying assumptions. First of all, we assume that

$$1 \gg \frac{D_t \nu_{ie}}{\omega_{ci}^2}. \quad (52)$$

This condition allows us to neglect the collision contribution on the left hand-side of Equations (40) and (45). We note that the relation between ω_{ci} and ν_{ie} in inequality (52) can be arbitrary because $D_t \ll \omega_{ci}$. For estimations of contribution of the different terms, we assume here and below that $D_t \gtrsim \omega_{bi,e}$. We also consider that

$$\max \left\{ \frac{D_t^2}{c_{si}^2} \frac{\partial}{\partial y} \left(\frac{\partial}{\partial z} \right)^{-2}; \frac{D_t}{\omega_{ci}} \frac{\partial}{\partial x} \right\} L \gg \frac{D_t \nu_{ie}}{\omega_{ci}^2} \frac{\partial^2}{\partial x \partial y} \left(\frac{\partial}{\partial z} \right)^{-2}, \quad (53)$$

where L is the typical inhomogeneity scale length along the x -axis. Under conditions (52) and (53), we can neglect the contribution of the collisional terms proportional to ν_{ie}^2 . Then solving Equations (40), (45), and (50), we can express the current \mathbf{j}_1 through \mathbf{E}_1

$$\begin{aligned} \frac{4\pi}{D_t} j_{1x} &= \varepsilon_{xx} E_{1x} - \varepsilon_{xy} E_{1y} - \varepsilon_{xz} E_{1z}, \\ \frac{4\pi}{D_t} j_{1y} &= -\varepsilon_{yx} E_{1x} + \varepsilon_{yy} E_{1y} + \varepsilon_{yz} E_{1z}, \\ \frac{4\pi}{D_t} j_{1z} &= \varepsilon_{zx} E_{1x} + \varepsilon_{zy} E_{1y} - \varepsilon_{zz} E_{1z}. \end{aligned} \quad (54)$$

The following notations are introduced here:

$$\begin{aligned} \varepsilon_{xx} &= a_{xx}, \varepsilon_{xy} = a_{xy} - \frac{D_t \nu_{ie}}{\omega_{pi}^2} \frac{a_{xz} (a_{zy} + a_{zy}^*)}{(1 - id_z)}, \\ \varepsilon_{xz} &= a_{xz} - \frac{D_t \nu_{ie}}{\omega_{pi}^2} \left[a_{xy} (a_{yz} - a_{yz}^*) - \frac{a_{xz} (a_{zz} + a_{zz}^*)}{(1 - id_z)} \right], \\ \varepsilon_{yx} &= a_{yx} + \frac{D_t \nu_{ie}}{\omega_{pi}^2} \frac{a_{yz} a_{zx}}{(1 - id_z)}, \varepsilon_{yy} = a_{yy} + a_{yy}^* - \frac{D_t \nu_{ie}}{\omega_{pi}^2} \frac{a_{yz} (a_{zy} + a_{zy}^*)}{(1 - id_z)}, \\ \varepsilon_{yz} &= a_{yz} - a_{yz}^* + \frac{D_t \nu_{ie}}{\omega_{pi}^2} \frac{a_{yz} (a_{zz} + a_{zz}^*)}{(1 - id_z)}, \\ \varepsilon_{zx} &= \frac{a_{zx}}{1 - id_z}, \varepsilon_{zy} = \frac{a_{zy} + a_{zy}^*}{1 - id_z}, \varepsilon_{zz} = \frac{a_{zz} + a_{zz}^*}{1 - id_z}, \end{aligned} \quad (55)$$

where

$$id_z = a_{zz} \frac{D_t \nu_{ie}^0}{\omega_{pi}^2}. \quad (56)$$

We keep in ε_{ij} ($i, j = x, y, z$) some terms, which will turn out to be small, because we do not know *a priori* solution of the dispersion relation.

8. DISPERSION RELATION

To derive the dispersion relation, we apply the Fourier transformation to the electromagnetic field and current, assuming that $\mathbf{E}_1 \sim \mathbf{E}_{1k} \exp(i\mathbf{k}\mathbf{r} - i\omega t)$, where $k = \{\mathbf{k}, \omega\}$. We take into account all three components of the wave vector, $\mathbf{k} = (k_x, k_y, k_z)$. Then using Equations (9), (10), and (54), we obtain

$$\hat{\mathbf{A}}_k \mathbf{E}_{1k} = \mathbf{0}, \quad (57)$$

where $\mathbf{E}_{1k} = (E_{1xk}, E_{1yk}, E_{1zk})$ and the matrix $\hat{\mathbf{A}}_k$ has the form

$$\hat{\mathbf{A}}_k = \begin{vmatrix} n_y^2 + n_z^2 - \varepsilon_{xx}, & -n_x n_y + \varepsilon_{xy}, & -n_x n_z + \varepsilon_{xz} \\ -n_x n_y + \varepsilon_{yx}, & n_x^2 + n_z^2 - \varepsilon_{yy}, & -n_y n_z - \varepsilon_{yz} \\ -n_x n_z - \varepsilon_{zx}, & -n_y n_z - \varepsilon_{zy}, & n_x^2 + n_y^2 + \varepsilon_{zz} \end{vmatrix}.$$

Here $\mathbf{n} = \mathbf{k}c/\omega$. We keep for values ε_{ij} the same notations as above. The dispersion relation is the determinant of the matrix $\hat{\mathbf{A}}_k$ equal to zero

$$\begin{aligned} 0 = & (n_y^2 + n_z^2 - \varepsilon_{xx}) [(n_x^2 + n_z^2 - \varepsilon_{yy}) \varepsilon_{zz} - \varepsilon_{zy} \varepsilon_{yz}] \\ & - (n_x n_y - \varepsilon_{yx}) [(n_x n_y - \varepsilon_{xy}) \varepsilon_{zz} - \varepsilon_{xz} \varepsilon_{zy} + \varepsilon_{zx} \varepsilon_{yz}]. \end{aligned} \quad (58)$$

When obtaining Equation (58), we have taken into account that according to Equations (41), (46), (51), and (55) inequalities $\varepsilon_{zz} \gg \varepsilon_{xx}, \varepsilon_{xy}, \varepsilon_{xz}, \varepsilon_{yy}, \varepsilon_{yz}, \varepsilon_{zy}$ and $\varepsilon_{xx} \varepsilon_{zz} \gg \varepsilon_{xz} \varepsilon_{zx}$ are satisfied. Expression $\varepsilon_{xz} \varepsilon_{zy} - \varepsilon_{zx} \varepsilon_{yz}$ can be omitted under the condition $1 \gg 1/(1 - id_z) kL$,

when $\Omega \gtrsim \omega$ or $T_i \neq T_e$ at $\Omega \ll \omega$. Here, the value k is equal to $k = |\mathbf{k}|$ and $\Omega = (\gamma - 1) \chi_{e0} k_z^2 / n_{e0}$ (see Eq. [35]).

To further simplify the dispersion relation (58), we neglect the collision contribution in ε_{xy} and ε_{yx} . The corresponding condition can be written in the form

$$1 \gg \frac{1}{(1 - id_z)} \left(id_z + \frac{\nu_{ie}}{\omega_{ci}} \right) \left(\frac{1}{kL} + \frac{\omega^2}{k_z^2 c_{si}^2} \right). \quad (59)$$

We remind that the ratio ν_{ie}/ω_{ci} in this inequality is arbitrary. Then Equation (58) takes the form

$$\omega^2 = k_z^2 c_A^2 + \frac{\omega_{ci}^2}{\omega_{pi}^2} D_t^2 \left(\varepsilon_{yy1} + \frac{\varepsilon_{zy} \varepsilon_{yz}}{\varepsilon_{zz}} \right) \frac{(k_y^2 + k_z^2)}{(k_x^2 + k_y^2 + k_z^2)}, \quad (60)$$

where $c_A = (B_0/4\pi m_i n_{i0})^{1/2}$ is the ion Alfvén velocity. The following notation is introduced here:

$$\varepsilon_{yy1} = \frac{\omega_{pi}^2}{\omega_{ci}^2} \left[\frac{\omega_{bi}^2}{D_t^2} + \frac{\omega_{be}^2}{D_t^2} \frac{\gamma D_t}{\gamma D_t + \Omega} \right] + a_{yy}^* - \frac{D_t \nu_{ie}}{\omega_{pi}^2} \frac{a_{yz} (a_{zy} + a_{zy}^*)}{(1 - id_z)}. \quad (61)$$

For convenience, we retain the symbol $D_t = -i\omega$ on the right hand-side of Equation (60). Below, we consider the dispersion relation in the collisionless as well as in the collisional cases.

8.1. Collisionless case

In this case, we assume that

$$d_z \ll 1, \quad (62)$$

where d_z is defined by expression (56). Under condition (62), the collisional term in ε_{yy1} is unimportant. Then the dispersion relation is given by

$$\omega^2 = k_z^2 c_A^2 + W c_s^2 \frac{(k_y^2 + k_z^2)}{(k_x^2 + k_y^2 + k_z^2)}, \quad (63)$$

where $c_s^2 = c_{si}^2 c_{se1}^2 / (c_{si}^2 + c_{se1}^2)$ and

$$W = \frac{1}{c_s^2} \left(\omega_{bi}^2 + \omega_{be}^2 \frac{\gamma D_t}{\gamma D_t + \Omega} + g_e \frac{\partial T_{e0}^*}{T_{e0} \partial x} \frac{\Omega}{\gamma D_t + \Omega} \right) + \left\{ -\frac{\omega_{bi}^2}{g_i} + \frac{1}{c_{se}^2} \left[(\gamma - 1) g_e \frac{\gamma D_t}{\gamma D_t + \Omega} + \frac{\partial c_{se}^2}{\partial x} \right] \right\} \left(\frac{g_i}{c_{si}^2} - \frac{g_e}{c_{se1}^2} \right). \quad (64)$$

The condition

$$1 \gg \frac{\nu_{ie}}{\omega_{ci}} \frac{k_y}{L k_z^2} \frac{\Omega}{(D_t + \Omega)}$$

has been supposed.

The case $\Omega \gg D_t$ In this limit, the value W is the following:

$$W = \frac{1}{c_s^2} \left(\omega_{bi}^2 + g_e \frac{\partial T_{e0}^*}{T_{e0} \partial x} \right) + \left(-\frac{\omega_{bi}^2}{g_i} + \frac{1}{c_{se}^2} \frac{\partial c_{se}^2}{\partial x} \right) \left(\frac{g_i}{c_{si}^2} - \frac{\gamma g_e}{c_{se}^2} \right).$$

This expression can be rewritten in the form

$$W = \gamma \frac{(g_i + g_e)}{c_{si}^2 c_{se}^2} \left[(\gamma - 1) g_i + \frac{1}{m_i} \left(\gamma \frac{\partial T_{i0}}{\partial x} + \frac{\partial T_{e0}}{\partial x} \right) \right], \quad (65)$$

where we have used expression (43) for $\omega_{bi,e}^2$. We see that instability, $W < 0$, is possible, if the temperature increases along the gravity (we assume that $g_{i,e}$ has the same sign as g).

The case $\Omega \ll D_t$ In the case $\Omega \ll D_i$, the thermal conductivity is absent. The value W is given by

$$W = \frac{1}{c_s^2} (\omega_{bi}^2 + \omega_{be}^2) - \left(\frac{\omega_{bi}^2}{g_i} - \frac{\omega_{be}^2}{g_e} \right) \left(\frac{g_i}{c_{si}^2} - \frac{g_e}{c_{se}^2} \right)$$

or

$$W = \frac{(g_i + g_e)}{c_{si}^2 c_{se}^2} \left[(\gamma - 1) (g_i + g_e) + \frac{\gamma}{m_i} \left(\frac{\partial T_{i0}}{\partial x} + \frac{\partial T_{e0}}{\partial x} \right) \right]. \quad (66)$$

Comparing Equations (65) and (66), we see that the thermal conductivity is not of fundamental importance.

8.2. Collisional case

We now assume that

$$id_z \gg 1. \quad (67)$$

In this case, we can neglect the term $\varepsilon_{zy}\varepsilon_{yz}/\varepsilon_{zz}$ in Equation (60). However, the collisional term in expression (61) gives the same contribution as other terms. As a result, we obtain again Equation (63) with W defined by Equation (64). Thus, the dispersion relation is the same for both the collisionless and collisional cases. We note that this result has also been obtained for the case in which gravity is parallel to the magnetic field (Nekrasov & Shadmehri 2010).

8.3. Polarization of the electric field perturbation

The dispersion relation (58) without $\varepsilon_{zx}\varepsilon_{yz} - \varepsilon_{xz}\varepsilon_{zy}$ can be obtained, if we neglect some terms in the matrix $\hat{\mathbf{A}}_k$

$$\hat{\mathbf{A}}_k = \begin{vmatrix} n_y^2 + n_z^2 - \varepsilon_{xx}, & -n_x n_y + \varepsilon_{xy}, & 0 \\ -n_x n_y + \varepsilon_{yx}, & n_x^2 + n_z^2 - \varepsilon_{yy}, & -\varepsilon_{yz} \\ 0 & -\varepsilon_{zy}, & \varepsilon_{zz} \end{vmatrix}.$$

Then Equation (57) takes the form

$$\begin{aligned} (n_y^2 + n_z^2 - \varepsilon_{xx}) E_{1xk} + (-n_x n_y + \varepsilon_{xy}) E_{1yk} &= 0, \\ (-n_x n_y + \varepsilon_{yx}) E_{1xk} + (n_x^2 + n_z^2 - \varepsilon_{yy}) E_{1yk} - \varepsilon_{yz} E_{1zk} &= 0, \\ -\varepsilon_{zy} E_{1yk} + \varepsilon_{zz} E_{1zk} &= 0. \end{aligned} \quad (68)$$

If we put $n_x = 0$, then the magnetosonic wave $\omega^2 = k_z^2 c_A^2$ is split, and Equation (60) describes the Alfvén type wave. When $n_y = 0$, the Alfvén wave $\omega^2 = k_z^2 c_A^2$ is split,

and Equation (60) describes the magnetosonic type wave. In the case $n_x = n_y = 0$, both waves are of the Alfvén type. In a general case, we have $\omega^2 \neq k_z^2 c_A^2$ and $E_{1xk} = [k_x k_y / (k_y^2 + k_z^2)] E_{1yk}$.

We see from the system of equations (68) that $E_{1zk} = (\varepsilon_{zy}/\varepsilon_{zz}) E_{1yk} \ll E_{1yk}$. However, the contribution of the longitudinal electric field E_{1zk} must be taken into account in the collisionless as well as in the collisional cases (see Eqs. [60] and [61]).

9. DISCUSSION

The dispersion relation (60) emphasizes the important role of the perturbed current and electric field along the background magnetic field. These perturbations produce the terms connected with ε_{yz} , ε_{zy} , and ε_{zz} in the dispersion. Taking into account these terms allows us to derive the correct dispersion relation which has the form (63). The value W , defined by expression (64), is available for both cases when the electron thermal conductivity is present or absent. Expressions (65) and (66) show that the thermal conductivity is not of fundamental importance for the buoyancy instability, if the latter can be excited. For the instability, the temperature gradient of ions and electrons must have the sign opposite to that of $g_{i,e}$. Under assumption that g_i and g_e have the same sign as g , the temperature must increase along the gravity for the instability to be excited.

The dispersion relation (63) takes into account collisions. Except for the anisotropic thermal conductivity adopted in this paper, the relation between ω_{ci} and ν_{ie} can be arbitrary in the framework of conditions (52), (53), and (59). Under these conditions, the dispersion relation for the collisionless (62) and collisional (67) cases is the same. This result has also been obtained for the case in which the background magnetic field and gravity are parallel to each other (Nekrasov & Shadmehri 2010).

From Equations (23) and (24), we can find the ion number density and pressure perturbations. Using Equations (38), (42), and (47), we can calculate the value $\nabla \cdot \mathbf{v}_{i1}$. Keeping the main terms, we have

$$\nabla \cdot \mathbf{v}_{i1} \simeq \frac{g_i}{\omega_{ci} c_{si}^2} F_{i1y} - \frac{D_t}{c_{si}^2} \left(\frac{\partial}{\partial z} \right)^{-1} F_{i1z}, \quad (69)$$

Substituting v_{i1x} and $\nabla \cdot \mathbf{v}_{i1}$ in Equation (23), we obtain an estimation

$$-D_t \frac{n_{i1}}{n_{i0}} \simeq \frac{1}{\omega_{ci}} \left(\frac{1}{n_{i0}} \frac{\partial n_{i0}}{\partial x} + \frac{g_i}{c_{si}^2} \right) F_{i1y} - \frac{D_t}{c_{si}^2} \left(\frac{\partial}{\partial z} \right)^{-1} F_{i1z}. \quad (70)$$

We see from Equations (69) and (70) that $\nabla \cdot \mathbf{v}_{i1} \sim D_t n_{i1}/n_{i0}$. From the system of equations (68), it is followed that $E_{1zk} = (\varepsilon_{zy}/\varepsilon_{zz}) E_{1yk}$. Thus, both terms on the right hand-side of equation (70) are of the same order. An estimation for the ion pressure perturbation is given by

$$D_t \frac{p_{i1}}{\gamma p_{i0}} \simeq \frac{D_t}{c_{si}^2} \left(\frac{\partial}{\partial z} \right)^{-1} F_{i1z}.$$

We see that $n_{i1}/n_{i0} \sim p_{i1}/p_{i0}$ due to the longitudinal electric field perturbation. The analogous conclusion can also be made for electrons. This result contradicts an assumption that $n_1/n_0 \gg p_1/p_0$ which one uses in the MHD analysis of buoyancy instabilities. We note that the ideal MHD does not involve the field E_z .

Let us discuss the relevance of conditions used in this paper to real astrophysical systems. As an example, we will consider an intracluster medium. However, our conditions have a more general applicability. Observations show that all astrophysical objects in cosmic space have magnetic fields of μG strength in galaxy clusters and molecular clouds (e.g., Carilli & Taylor 2002; Wardle & Ng 1999) to G and more in accretion disks (e.g., Desch 2004; Donati et al. 2005). For such magnetic fields, the ion Larmor radius ρ_i is considerably smaller than the typical inhomogeneity length L in these objects, i.e. $\rho_i \ll L$. For example, if we take for ICM $B_0 \sim 1 \mu\text{G}$ and $T_i \sim 1 \text{ keV}$, we obtain $\rho_i \sim 2 \times 10^4 \text{ km}$.

The magnitude of L is \sim tens of kpc. In ICM, the collision frequency ν_{ie} is much less than the ion cyclotron frequency ω_{ci} , $\nu_{ie} \ll \omega_{ci}$. In dense molecular clouds and accretion disks, the ions can be unmagnetized, $\nu_{ie} \gg \omega_{ci}$ (Wardle & Ng 1999). However, many conditions of our consideration (see below) can also be satisfied in the last case. For example, this relates to inequality (17) justifying solutions (13) and (14) in the equilibrium state. The electromagnetic buoyancy perturbations have a dynamical frequency ω and wavelength λ much less than the ion cyclotron and sound frequencies and inhomogeneity scale length, respectively. Thus, the conditions (52) and (59) are satisfied for a weakly collisional plasmas as ICM and can be satisfied for a strong collisional plasma when $\nu_{ie} \gtrsim \omega_{ci}$. Accounting for the fact that $|\omega| \sim g/c_s$ (indices i and e are omitted), we can treat the medium as a stationary one (see Eq. [18]), if $1 \gg (\nu_{ie}/\omega_{ci})(\rho_i/L)$ that is justified for astrophysical objects. The condition (20) is satisfied for the last inequality (which also relates to Equation (28)) and under $\rho_i \ll L$ in the case $v_{i0y} \neq \text{const.}$ The condition (53) can be written in the form $1 \gg (\nu_{ie}/\omega_{ci})(\lambda/L; \rho_i/\lambda)$, where $\lambda \ll L$ and $\rho_i \ll \lambda$. The first condition (37) is the following: $1 \gg (\rho_i/\lambda)(1; \nu_{ie}/\omega_{ci})$. It is also true for the transverse component of the second condition (37). However, taking into account that $E_{1zk} = (\varepsilon_{zy}/\varepsilon_{zz}) E_{1yk}$ (see Sec. 8.3), both sides of the z -component of the second inequality (37) can be of the same order, if $\mathbf{v}_{i,e0} \neq \mathbf{0}$. Owing to indefiniteness of the background velocities also containing the electric field, we here do not consider their effect. Nevertheless, the streaming instabilities can also take place.

In our analysis, we consider for generality that ions and electrons have the different temperatures. However, in Eqs. (3), (6), and (7), the terms describing the energy exchange between species due to their collisions have not been taken into account. This is possible, if the dynamical time scale is smaller than the time scale of smoothing of the ion and electron temperatures, i.e. $\nu_{ie} \ll \omega$. In the opposite case, $\nu_{ie} \gg \omega$, the perturbed temperatures of electrons and ions are almost equal each other. Equations (6) and (7) for electrons will keep

their form because $\mathbf{v}_{e1} \approx \mathbf{v}_{i1}$. In the case $T_{e0} \approx T_{i0}$, these equations will stay the same with the heat flux two times less than the former one. Equation for the ion temperature will not be needed.

10. IMPLICATIONS OF THE OBTAINED RESULTS FOR GALAXY CLUSTERS

According to a simplified point of view, the core of cluster must be cooled. In fact, observations show that in most clusters the cooling time-scale near the center of cluster, 10^8 to 10^9 yr, is much shorter than a cluster's age 10^{10} yr (e.g., Fabian 1994; Peres et al. 1998; Allen 2000). But such high rates of cooling has been definitely ruled out by the X-ray observations showing that the cluster core is sufficiently hot (e.g., Allen 2000). Thus, some heating mechanisms are in operation, though we have little knowledge about them. Different heating mechanisms from AGN feedback to cosmic rays and turbulence are proposed to resolve the "cooling flow problem" (e.g., Eilek 2004; Binney & Tabor 1995; Loewenstein et al. 1991; Reynolds 2002). Exactly for this reason, a study of buoyancy instabilities in ICM has a purpose to find a solution of this longstanding problem in clusters of galaxies. The plasma in ICM is turbulent (e.g., Loewenstein & Fabian 1990; Cattaneo & Teyssier 2007). Buoyancy instabilities could be one of possible sources of turbulence resulting in the emergence of heat fluxes along the magnetic field in the direction of core. Thus, the latter could be heated. The total nonlinear picture of this process which can include reorientation of the magnetic field (Parrish & Stone 2007) is very complex for the analytical consideration and can only be investigated numerically.

In the framework of the linear ideal MHD, new buoyancy instabilities have been found, when the thermal conduction is the dominant mode of heat transport (Balbus 2000, 2001;

Quataert 2008). In the present paper, in the framework of the multicomponent **E**-approach, we have found generalized growth rates (for the same geometry as considered by Balbus 2000) for both the negligible and dominant thermal conduction. In both cases, the growth rates have the same order of magnitude. However, our modified conditions of instability are different from the MHD case because of the multifluid nature of the system (see Section 9). We have shown that conditions for the buoyancy instability have the form which is analogous to the Schwarzschild criterion (Schwarzschild 1958). Thus, the ICM plasma can be buoyantly unstable for both large and small thermal conductivity. This increases the range of wavelengths of the unstable perturbations because the thermal flux is proportional to the wave number. Our linear analysis shows that the multifluid nature of plasma in the ICM and other astrophysical objects can not be neglected when one investigates heat flows as a result of buoyancy instabilities. We also note that buoyancy instabilities are of the electromagnetic nature. Therefore, they can contribute to the magnetic field activity in astrophysical objects.

11. CONCLUSION

In this paper, we have investigated buoyancy instabilities in the magnetized electron-ion astrophysical plasmas in which the background magnetic field and gravity are perpendicular to each other. We have applied the multicomponent **E**-approach in which the dynamical equations for ions and electrons are solved separately via the electric field perturbations. The perturbed current and Faraday’s and Ampere’s laws have been used to derive the dispersion relation. We have included collisions between electrons and ions. Except for the anisotropy of the electron heat flux adopted in this paper, in other respects, the relation between the ion-electron collision frequency and ion cyclotron frequency can be arbitrary

in the framework of approximations, which have been made. The important role of the longitudinal electric field perturbations, which are not captured by the MHD equations, has been shown. The obtained growth rates for cases of strong and weak electron thermal conductivity show that an instability is possible when the temperature gradients of ions and electrons are directed along the gravity. We have shown that the relative perturbations of number density and pressure are of the same order as a result of action of the longitudinal electric field perturbation.

Results obtained in this paper are applicable to the magnetized collisional stratified objects and can be useful for a search of sources of turbulent transport of energy and matter in the ICM and other astrophysical objects.

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APPENDIX A

A. SOLUTION OF EQUATION (22)

The components of Equation (22) are the following:

$$D_{ti}v_{i1x} = -\frac{1}{m_i n_{i0}} \frac{\partial p_{i1}}{\partial x} - g_i \frac{n_{i1}}{n_{i0}} + G_{i1x} + \omega_{ci} v_{i1y}, \quad (\text{A1})$$

$$D_{ti}v_{i1y} = -\frac{1}{m_i n_{i0}} \frac{\partial p_{i1}}{\partial y} + G_{i1y} - C_{i1y} - \omega_{ci} v_{i1x}, \quad (\text{A2})$$

$$D_{ti}v_{i1z} = -\frac{1}{m_i n_{i0}} \frac{\partial p_{i1}}{\partial z} + G_{i1z}. \quad (\text{A3})$$

Let us apply the operator D_{ti} to Equation (A1). We do not differentiate the value $1/n_{i0}$ and change $D_{ti}\partial p_{i1}/\partial x$ by $\partial D_{ti}p_{i1}/\partial x$ according to condition (20). Then we use Equations (A2), (23) and (24). As a result, we obtain the following equation connecting v_{i1x} and $\nabla \cdot \mathbf{v}_{i1}$:

$$\begin{aligned} \left[D_{ti} (D_{ti}^2 + \omega_{ci}^2) + g_i \left(\omega_{ci} \frac{\partial}{\partial y} + D_{ti} \frac{\partial}{\partial x} \right) \right] v_{i1x} &= D_{ti} [D_{ti} G_{i1x} + \omega_{ci} (G_{i1y} - C_{i1y})] \\ &+ \left\{ D_{ti} \left[g_i (1 - \gamma) + c_{si}^2 \frac{\partial}{\partial x} \right] + \omega_{ci} c_{si}^2 \frac{\partial}{\partial y} \right\} \nabla \cdot \mathbf{v}_{i1}, \end{aligned} \quad (\text{A4})$$

where $c_{si} = (\gamma p_{i0}/m_i n_{i0})^{1/2}$ is the ion sound velocity. To obtain the second equation expressing $\nabla \cdot \mathbf{v}_{i1}$ through v_{i1x} , we apply the operator $D_{ti}\partial/\partial y$ to Equation (A2), $D_{ti}\partial/\partial z$ to Equation (A3) and add the resulting equations. Then using Equation (24), we find

$$\begin{aligned} \left[D_{ti}^2 - c_{si}^2 \left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \right] \nabla \cdot \mathbf{v}_{i1} &= D_{ti} \left[\frac{\partial}{\partial z} G_{i1z} + \frac{\partial}{\partial y} (G_{i1y} - C_{i1y}) \right] \\ &+ \left[D_{ti} \left(D_{ti} \frac{\partial}{\partial x} - \omega_{ci} \frac{\partial}{\partial y} \right) - g_i \left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \right] v_{i1x}. \end{aligned} \quad (\text{A5})$$

We introduce the following notations:

$$\begin{aligned} K_1 &= \left[D_{ti} (D_{ti}^2 + \omega_{ci}^2) + g_i \left(\omega_{ci} \frac{\partial}{\partial y} + D_{ti} \frac{\partial}{\partial x} \right) \right], \\ K_2 &= \left\{ D_{ti} \left[g_i (1 - \gamma) + c_{si}^2 \frac{\partial}{\partial x} \right] + \omega_{ci} c_{si}^2 \frac{\partial}{\partial y} \right\}, \\ M_{i1x} &= D_{ti} [D_{ti} G_{i1x} + \omega_{ci} (G_{i1y} - C_{i1y})]. \end{aligned} \quad (\text{A6})$$

Then Equation (A4) takes the form

$$K_1 v_{i1x} = M_{i1x} + K_2 \nabla \cdot \mathbf{v}_{i1}. \quad (\text{A7})$$

We further apply the operator K_1 to Equation (A5) and use Equation (A7). As a result, we find equation for $\nabla \cdot \mathbf{v}_{i1}$

$$\begin{aligned} 0 &= K_1 D_{ti} \left[\frac{\partial}{\partial z} G_{i1z} + \frac{\partial}{\partial y} (G_{i1y} - C_{i1y}) \right] + \left[D_{ti} \left(D_{ti} \frac{\partial}{\partial x} - \omega_{ci} \frac{\partial}{\partial y} \right) - g_i \left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \right] M_{i1x} \\ &+ \left\{ \left[D_{ti} \left(D_{ti} \frac{\partial}{\partial x} - \omega_{ci} \frac{\partial}{\partial y} \right) - g_i \left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \right] K_2 - K_1 \left[D_{ti}^2 - c_{si}^2 \left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \right] \right\} \nabla \cdot \mathbf{v}_{i1}. \end{aligned} \quad (\text{A8})$$

If we consider Equation (A8) without electromagnetic forces and the background magnetic field, we will obtain equation

$$H_i \nabla \cdot \mathbf{v}_{i1} = 0,$$

where

$$\begin{aligned} H_i = & -D_{ti}^4 + D_{ti}^2 c_{si}^2 \left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial x^2} \right) + g_i \left[(\gamma - 1) g_i + \frac{\partial c_{si}^2}{\partial x} \right] \left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \\ & + D_{ti}^2 \left(\frac{\partial c_{si}^2}{\partial x} - \gamma g_i \right) \frac{\partial}{\partial x}. \end{aligned}$$

This equation describe the ion sound and internal (ion) gravity waves. When obtaining Equation (A8), we have excluded v_{i1x} . We also could exclude $\nabla \cdot \mathbf{v}_{i1}$. In this case, the term $\partial c_{si}^2 / \partial x$ in the last term in the expression for H_i would have the sign $-$. It is connected with the form of dependence of v_{i1x} from $\nabla \cdot \mathbf{v}_{i1}$ and vice versa in Equations (A4) and (A5).

Let us introduce the following notations:

$$\begin{aligned} N_{i1x} &= K_1 D_{ti} \left[\frac{\partial}{\partial z} G_{i1z} + \frac{\partial}{\partial y} (G_{i1y} - C_{i1y}) \right] + \left[D_{ti} \left(D_{ti} \frac{\partial}{\partial x} - \omega_{ci} \frac{\partial}{\partial y} \right) - g_i \left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \right] M_{i1x}, \\ K_3 &= \left\{ \left[D_{ti} \left(D_{ti} \frac{\partial}{\partial x} - \omega_{ci} \frac{\partial}{\partial y} \right) - g_i \left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \right] K_2 - K_1 \left[D_{ti}^2 - c_{si}^2 \left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \right] \right\}. \end{aligned} \quad (\text{A9})$$

Then Equation (A8) is given by

$$K_3 \nabla \cdot \mathbf{v}_{i1} = -N_{i1x}. \quad (\text{A10})$$

Using notations (A6), we can represent the operator K_3 and the value N_{i1x} defined by Equations (A9) in the form

$$\begin{aligned} K_3 = & \omega_{ci}^2 D_{ti} \left(c_{si}^2 \frac{\partial^2}{\partial z^2} - D_{ti}^2 \right) + \omega_{ci} D_{ti}^2 \left[(\gamma - 2) g_i + \frac{\partial c_{si}^2}{\partial x} \right] \frac{\partial}{\partial y} \\ & + D_{ti}^3 \left[c_{si}^2 \left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial x^2} \right) - D_{ti}^2 \right] + g_i D_{ti} \left[(\gamma - 1) g_i + \frac{\partial c_{si}^2}{\partial x} \right] \left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \\ & + D_{ti}^3 \left(-\gamma g_i + \frac{\partial c_{si}^2}{\partial x} \right) \frac{\partial}{\partial x} \end{aligned} \quad (\text{A11})$$

and

$$\begin{aligned}
\frac{1}{D_{ti}} N_{i1x} = & D_{ti} \left[D_{ti} \left(D_{ti} \frac{\partial}{\partial x} - \omega_{ci} \frac{\partial}{\partial y} \right) - g_i \left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \right] G_{i1x} \\
& + \left[D_{ti}^2 \left(D_{ti} \frac{\partial}{\partial y} + \omega_{ci} \frac{\partial}{\partial x} \right) + g_i \left(D_{ti} \frac{\partial^2}{\partial x \partial y} - \omega_{ci} \frac{\partial^2}{\partial z^2} \right) \right] (G_{i1y} - C_{i1y}) \\
& + \left[D_{ti} (D_{ti}^2 + \omega_{ci}^2) + g_i \left(\omega_{ci} \frac{\partial}{\partial y} + D_{ti} \frac{\partial}{\partial x} \right) \right] \frac{\partial}{\partial z} G_{i1z}.
\end{aligned} \tag{A12}$$

We further assume the following simplifications ($g_i \sim \partial c_{si}^2 / \partial x$):

$$\begin{aligned}
\omega_{ci}^2 & \gg c_{si}^2 \left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial x^2} \right); D_{ti}^2; g_i \frac{\partial}{\partial x}, \\
D_{ti} & \gg \frac{g_i}{\omega_{ci}} \frac{\partial}{\partial y}, D_{ti}^2 \gg \frac{g_i^2}{\omega_{ci}^2} \frac{\partial^2}{\partial z^2}.
\end{aligned} \tag{A13}$$

Now, $D_{ti} = \partial / \partial t + v_{i0x} \partial / \partial x$. Then the operator K_3 takes the simple form (see Eq. [A11])

$$K_3 = \omega_{ci}^2 D_{ti} \left(c_{si}^2 \frac{\partial^2}{\partial z^2} - D_{ti}^2 \right) + \omega_{ci} D_{ti}^2 \left[(\gamma - 2) g_i + \frac{\partial c_{si}^2}{\partial x} \right] \frac{\partial}{\partial y}. \tag{A14}$$

We take into account the last small term on the right hand-side of Equation (A14) to obtain some additional terms proportional to ω_{ci}^{-2} in expressions for components of \mathbf{v}_{i1} .

A.1. The velocity v_{i1x}

From Equations (A4), (A10), (A12), and (A14), we find the velocity v_{i1x} , using conditions (A13),

$$\begin{aligned}
v_{i1x} = & \frac{D_{ti} \left(c_{si}^2 \frac{\partial^2}{\partial y^2} + c_{si}^2 \frac{\partial^2}{\partial z^2} - D_{ti}^2 \right)}{\omega_{ci}^2 \left(c_{si}^2 \frac{\partial^2}{\partial z^2} - D_{ti}^2 \right)} G_{i1x} + \frac{1}{\omega_{ci}} (G_{i1y} - C_{i1y}) \\
& + \frac{D_{ti}}{\omega_{ci}^2} \frac{1}{\left(c_{si}^2 \frac{\partial^2}{\partial z^2} - D_{ti}^2 \right)} \left(g_i - c_{si}^2 \frac{\partial}{\partial x} \right) \frac{\partial}{\partial y} (G_{i1y} - C_{i1y}) - \frac{c_{si}^2}{\omega_{ci} \left(c_{si}^2 \frac{\partial^2}{\partial z^2} - D_{ti}^2 \right)} \frac{\partial^2}{\partial y \partial z} G_{i1z} \\
& - \frac{D_{ti}}{\omega_{ci}^2} \left[(1 - \gamma) g_i + c_{si}^2 \frac{\partial}{\partial x} \right] \frac{1}{\left(c_{si}^2 \frac{\partial^2}{\partial z^2} - D_{ti}^2 \right)} \frac{\partial}{\partial z} G_{i1z} + \frac{D_{ti} c_{si}^2 \left[(\gamma - 2) g_i + \frac{\partial c_{si}^2}{\partial x} \right]}{\omega_{ci}^2 \left(c_{si}^2 \frac{\partial^2}{\partial z^2} - D_{ti}^2 \right)^2} \frac{\partial^3}{\partial y^2 \partial z} G_{i1z}.
\end{aligned} \tag{A15}$$

When obtaining solution (A15), we have used some additional conditions

$$\begin{aligned}
 \left(c_{si}^2 \frac{\partial^2}{\partial y^2} + c_{si}^2 \frac{\partial^2}{\partial z^2} - D_{ti}^2 \right) &\gg \frac{D_{ti}}{\omega_{ci}} c_{si}^2 \frac{\partial^2}{\partial x \partial y}, \\
 \left(g_i - c_{si}^2 \frac{\partial}{\partial x} \right) \frac{\partial}{\partial y} &\gg \frac{g_i^2}{D_{ti} \omega_{ci}} \frac{\partial^2}{\partial z^2}; \frac{D_{ti}}{\omega_{ci}} c_{si}^2 \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right); \frac{D_{ti}^3}{\omega_{ci}}, \\
 D_{ti} &\gg \frac{g_i}{\omega_{ci}} \left(\frac{\partial}{\partial y} \right)^{-1} \left(\frac{\partial}{\partial z} \right)^2.
 \end{aligned} \tag{A16}$$

We note that in general expressions, we do not use the local approximation.

A.2. The velocity v_{i1y}

The velocity v_{i1y} , we find from Equation (A1). Applying the operator D_{ti} to this equation and using Equations (23) and (24), we obtain

$$D_{ti}^2 v_{i1x} + g_i \frac{\partial v_{i1x}}{\partial x} = D_{ti} G_{i1x} + D_{ti} \omega_{ci} v_{i1y} + \left[g_i (1 - \gamma) + c_{si}^2 \frac{\partial}{\partial x} \right] \nabla \cdot \mathbf{v}_{i1}. \tag{A17}$$

From Equations (A10), (A12), (A14), (A15), and (A17), we find, using conditions (A13) and (A16),

$$\begin{aligned}
 v_{i1y} = & -\frac{1}{\omega_{ci}} G_{i1x} - \frac{D_{ti}}{\omega_{ci}^2} \left[(1 - \gamma) g_i + c_{si}^2 \frac{\partial}{\partial x} \right] \frac{1}{(c_{si}^2 \frac{\partial^2}{\partial z^2} - D_{ti}^2)} \frac{\partial}{\partial y} G_{i1x} \\
 & + \frac{D_{ti}}{\omega_{ci}^2} \left[1 + \frac{c_{si}^2 \frac{\partial^2}{\partial x^2}}{(c_{si}^2 \frac{\partial^2}{\partial z^2} - D_{ti}^2)} \right] (G_{i1y} - C_{ie}) \\
 & - \frac{D_{ti}}{\omega_{ci}^2} \frac{1}{(c_{si}^2 \frac{\partial^2}{\partial z^2} - D_{ti}^2)} \left[\gamma g_i + \frac{c_{si}^2 \frac{\partial^2}{\partial z^2}}{(c_{si}^2 \frac{\partial^2}{\partial z^2} - D_{ti}^2)} \frac{\partial c_{si}^2}{\partial x} \right] \frac{\partial}{\partial x} (G_{i1y} - C_{ie}) \\
 & + \frac{1}{\omega_{ci}^2 D_{ti}} \frac{g_i}{(c_{si}^2 \frac{\partial^2}{\partial z^2} - D_{ti}^2)} \left[(\gamma - 1) g_i + \frac{c_{si}^2 \frac{\partial^2}{\partial z^2}}{(c_{si}^2 \frac{\partial^2}{\partial z^2} - D_{ti}^2)} \frac{\partial c_{si}^2}{\partial x} \right] \frac{\partial^2}{\partial z^2} (G_{i1y} - C_{ie}) \\
 & + \frac{1}{\omega_{ci}} \left[(1 - \gamma) g_i + c_{si}^2 \frac{\partial}{\partial x} \right] \frac{1}{(c_{si}^2 \frac{\partial^2}{\partial z^2} - D_{ti}^2)} \frac{\partial}{\partial z} G_{i1z} \\
 & - \frac{1}{D_{ti} \omega_{ci}^2} \left\{ D_{ti}^2 + \frac{g_i}{c_{si}^2} \left[(\gamma - 1) g_i + \frac{\partial c_{si}^2}{\partial x} \right] \right\} \frac{c_{si}^2}{(c_{si}^2 \frac{\partial^2}{\partial z^2} - D_{ti}^2)} \frac{\partial^2}{\partial y \partial z} G_{i1z}
 \end{aligned} \tag{A18}$$

$$-\frac{D_{ti}}{\omega_{ci}^2} \left[g_i (1 - \gamma) + c_{si}^2 \frac{\partial}{\partial x} \right] \frac{\left[(\gamma - 2) g_i + \frac{\partial c_{si}^2}{\partial x} \right]}{\left(c_{si}^2 \frac{\partial^2}{\partial z^2} - D_{ti}^2 \right)^2} \frac{\partial^2}{\partial y \partial z} G_{i1z}.$$

A.3. The velocity v_{i1z}

The velocity v_{i1z} , we find from Equation (A3). Applying the operator D_{ti} and using Equation (24), we obtain

$$D_{ti}^2 v_{i1z} = -g_i \frac{\partial}{\partial z} v_{i1x} + D_{ti} G_{i1z} + c_{si}^2 \frac{\partial}{\partial z} \nabla \cdot \mathbf{v}_{i1}. \quad (\text{A19})$$

From Equations (A10), (A12), (A14), (A15), and (A19), we find, using conditions given above,

$$\begin{aligned} v_{i1z} = & \frac{1}{\omega_{ci}} \frac{c_{si}^2}{\left(c_{si}^2 \frac{\partial^2}{\partial z^2} - D_{ti}^2 \right)} \frac{\partial^2}{\partial y \partial z} G_{i1x} + \frac{D_{ti}}{\omega_{ci}^2} \frac{1}{\left(c_{si}^2 \frac{\partial^2}{\partial z^2} - D_{ti}^2 \right)} \left(g_i - c_{si}^2 \frac{\partial}{\partial x} \right) \frac{\partial}{\partial z} G_{i1x} \\ & - \frac{D_{ti}}{\omega_{ci}^2} \frac{c_{si}^2 \left[(\gamma - 2) g_i + \frac{\partial c_{si}^2}{\partial x} \right]}{\left(c_{si}^2 \frac{\partial^2}{\partial z^2} - D_{ti}^2 \right)^2} \frac{\partial^3}{\partial y^2 \partial z} G_{i1x} + \frac{1}{\omega_{ci}} \frac{1}{\left(c_{si}^2 \frac{\partial^2}{\partial z^2} - D_{ti}^2 \right)} \left(g_i - c_{si}^2 \frac{\partial}{\partial x} \right) \frac{\partial}{\partial z} (G_{i1y} - C_{i1y}) \\ & - \frac{1}{\omega_{ci}^2 D_{ti}} \frac{c_{si}^2}{\left(c_{si}^2 \frac{\partial^2}{\partial z^2} - D_{ti}^2 \right)} \left[D_{ti}^2 + \omega_{bi}^2 \frac{c_{si}^2 \frac{\partial^2}{\partial z^2}}{\left(c_{si}^2 \frac{\partial^2}{\partial z^2} - D_{ti}^2 \right)} - \frac{g_i^2}{c_{si}^2} \frac{D_{ti}^2}{\left(c_{si}^2 \frac{\partial^2}{\partial z^2} - D_{ti}^2 \right)} \right] \frac{\partial^2}{\partial y \partial z} (G_{i1y} - C_{i1y}) \\ & + \frac{D_{ti}}{\omega_{ci}^2} \frac{c_{si}^2 \left[(\gamma - 2) g_i + \frac{\partial c_{si}^2}{\partial x} \right]}{\left(c_{si}^2 \frac{\partial^2}{\partial z^2} - D_{ti}^2 \right)^2} \frac{\partial^3}{\partial x \partial y \partial z} (G_{i1y} - C_{i1y}) \\ & - \frac{D_{ti}}{\left(c_{si}^2 \frac{\partial^2}{\partial z^2} - D_{ti}^2 \right)} G_{i1z} + \frac{1}{\omega_{ci}} \frac{c_{si}^2 \left[(\gamma - 2) g_i + \frac{\partial c_{si}^2}{\partial x} \right]}{\left(c_{si}^2 \frac{\partial^2}{\partial z^2} - D_{ti}^2 \right)^2} \frac{\partial^3}{\partial y \partial z^2} G_{i1z} \\ & - \frac{1}{\omega_{ci}^2 D_{ti}} \frac{g_i}{\left(c_{si}^2 \frac{\partial^2}{\partial z^2} - D_{ti}^2 \right)} \left[(\gamma - 1) g_i + \frac{c_{si}^2 \frac{\partial^2}{\partial z^2}}{\left(c_{si}^2 \frac{\partial^2}{\partial z^2} - D_{ti}^2 \right)} \frac{\partial c_{si}^2}{\partial x} \right] \frac{\partial^2}{\partial z^2} G_{i1z}. \end{aligned} \quad (\text{A20})$$

Note that the small term proportional to $\omega_{ci}^{-1} G_{i1z}$ in Equation (A20) has appeared due to the small term in Equation (A14).

APPENDIX B

B. SOLUTION OF EQUATION (27)

The components of Equation (27) are the following:

$$0 = -\frac{1}{n_{e0}} \frac{\partial p_{e1}}{\partial x} - m_i g_e \frac{n_{e1}}{n_{e0}} + G_{e1x} + m_e \omega_{ce} v_{e1y}, \quad (\text{B1})$$

$$0 = -\frac{1}{n_{e0}} \frac{\partial p_{e1}}{\partial y} + G_{e1y} - C_{e1y} - m_e \omega_{ce} v_{e1x}, \quad (\text{B2})$$

$$0 = -\frac{1}{n_{e0}} \frac{\partial p_{e1}}{\partial z} + G_{e1z}. \quad (\text{B3})$$

Below, we find the components of \mathbf{v}_{e1} .

B.1. The velocity v_{e1x}

The velocity v_{e1x} can be easily found from Equations (B2) and (B3). Differentiating Equation (B2) over $\partial/\partial z$ and (B3) over $\partial/\partial y$ and subtracting one equation from another, we obtain

$$m_e \omega_{ce} \frac{\partial v_{e1x}}{\partial z} = \frac{\partial}{\partial z} (G_{e1y} - C_{e1y}) - \frac{\partial}{\partial y} G_{e1z}. \quad (\text{B4})$$

B.2. The value $\nabla \cdot \mathbf{v}_{e1}$

We can find the value $\nabla \cdot \mathbf{v}_{e1}$ from Equation (B2). Applying to this equation operator D_{te} and using Equations (36) and (B4), we obtain

$$\begin{aligned} c_{se}^2 \frac{(\gamma D_{te} + \Omega)}{\gamma (D_{te} + \Omega)} \nabla \cdot \mathbf{v}_{e1} &= \left[g_e + \frac{\Omega}{(D_{te} + \Omega)} \frac{\partial T_{e0}}{m_i \partial x} \right] v_{e1x} \\ &\quad - \frac{\partial T_{e0}}{m_i \partial x} \frac{1}{B_0} \frac{D_{te} \Omega}{(D_{te} + \Omega)} \left(\frac{\partial}{\partial z} \right)^{-1} B_{1x} - \frac{1}{m_i} D_{te} \left(\frac{\partial}{\partial z} \right)^{-1} G_{e1z}. \end{aligned} \quad (\text{B5})$$

B.3. The velocity v_{e1y}

We calculate the velocity v_{e1y} from Equation (B1), applying the operator D_{te} and using Equations (28) and (36). We further insert in the equation obtained the value $\nabla \cdot \mathbf{v}_{e1}$. The important point is to differentiate carefully the background electron number density and pressure. Then we use Equation (B4) for v_{e1x} . As a result of calculations, we obtain

$$\begin{aligned}
 m_e \omega_{ce} v_{e1y} = & -G_{e1x} + \frac{g_e}{c_{se}^2} \left[(\gamma - 1) g_e + \frac{\partial c_{se}^2}{\partial x} \right] \frac{\gamma}{(\gamma D_{te} + \Omega)} \frac{m_i}{m_e \omega_{ce}} (G_{e1y} - C_{e1y}) \quad (B6) \\
 & + \frac{\partial}{\partial x} \left(\frac{\partial}{\partial z} \right)^{-1} G_{e1z} - \frac{1}{c_{se}^2} \left[(\gamma - 1) g_e \frac{\gamma D_{te}}{(\gamma D_{te} + \Omega)} + \frac{\partial c_{se}^2}{\partial x} \right] \left(\frac{\partial}{\partial z} \right)^{-1} G_{e1z} \\
 & - \frac{g_e}{c_{se}^2} \left[(\gamma - 1) g_e + \frac{\partial c_{se}^2}{\partial x} \right] \frac{\gamma}{(\gamma D_{te} + \Omega)} \frac{m_i}{m_e \omega_{ce}} \frac{\partial}{\partial y} \left(\frac{\partial}{\partial z} \right)^{-1} G_{e1z} \\
 & + m_i g_e \frac{\partial T_{e0}}{T_{e0} \partial x} \frac{1}{B_0} \frac{\Omega}{(\gamma D_{te} + \Omega)} \left(\frac{\partial}{\partial z} \right)^{-1} B_{1x}.
 \end{aligned}$$

B.4. The velocity v_{e1z}

We find this velocity from $\nabla \cdot \mathbf{v}_{e1}$

$$\frac{\partial v_{e1z}}{\partial z} = \nabla \cdot \mathbf{v}_{e1} - \frac{\partial v_{e1x}}{\partial x} - \frac{\partial v_{e1y}}{\partial y}. \quad (B7)$$

Using solutions (B4)-(B6), we obtain from Equation (B7)

$$\begin{aligned}
 \frac{\partial v_{e1z}}{\partial z} = & \frac{1}{m_e \omega_{ce}} \frac{\partial}{\partial y} G_{e1x} - \frac{1}{m_e \omega_{ce}} \frac{\partial}{\partial x} (G_{e1y} - C_{e1y}) \quad (B8) \\
 & + \frac{1}{c_{se1}^2} \left[g_e + \frac{\Omega}{(D_{te} + \Omega)} \frac{\partial T_{e0}}{m_i \partial x} \right] \frac{1}{m_e \omega_{ce}} (G_{e1y} - C_{e1y}) \\
 & - m_i \frac{g_e}{c_{se}^2} \left[(\gamma - 1) g_e + \frac{\partial c_{se}^2}{\partial x} \right] \frac{\gamma}{(\gamma D_{te} + \Omega)} \left(\frac{1}{m_e \omega_{ce}} \right)^2 \frac{\partial}{\partial y} (G_{e1y} - C_{e1y}) \\
 & - \frac{1}{c_{se1}^2 m_i} D_{te} \left(\frac{\partial}{\partial z} \right)^{-1} G_{e1z} - \frac{\partial T_{e0}}{c_{se1}^2 m_i \partial x} \frac{1}{B_0} \frac{D_{te} \Omega}{(D_{te} + \Omega)} \left(\frac{\partial}{\partial z} \right)^{-1} B_{1x}.
 \end{aligned}$$

The velocity c_{se1} is defined by expression (49). When obtaining Equation (B8), we have used condition $D_{te} \gg g_e k_y / \omega_{ci}$. In this case, $D_{te} = \partial / \partial t + v_{e0x} \partial / \partial x$.